Regular Paper

論文テンプレート (日本語 UTF-8) - Article Title - Submission to Mechanized Mathematics and Its Applications, Works in Progresss

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Abstract

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1 Introduction

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$$D_{coll} = \frac{D_f + \frac{[S]^2}{K_D S_T} D_S}{1 + \frac{[S]^2}{K_D S_T}}, D_{sm} = \frac{D_f + \frac{[S]}{K_D} D_S}{1 + \frac{[S]}{K_D}}, \tag{1}$$

2 Methods

2.1 Etiam eget sapien nibh.

Nulla mi mi, Fig. 1 venenatis sed ipsum varius, volutpat euismod diam. Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. Fusce fringilla erat porttitor lectus cursus, [GROUP_17] vel sagittis arcu lobortis. Aliquam in enim semper, aliquam massa id, cursus neque. Praesent faucibus semper libero.

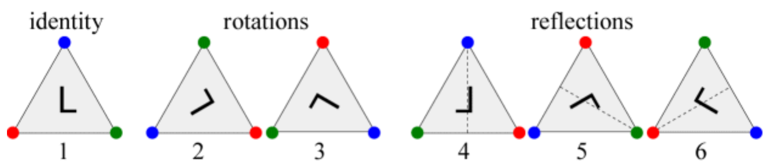

図 1. **Figure Title first bold sentence Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam.** Figure Caption Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. A: Lorem ipsum dolor sit amet. B: Consectetur adipiscing elit.

2.2 Formalizations

Nulla mi mi, Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. See Listing 1.

Listing 1. GROUP_17 - Th.16

```
theorem :: GROUP_17:16
  for G being finite commutative Group,
  h,k be Nat
  st card G = h*k & h,k are_coprime holds
  ex H,K being strict finite Subgroup of G st
  the carrier of H = {x where x is Element of G: x|^h = 1_G} & :: 注釈を入れてもよい
  the carrier of K = {x where x is Element of G: x|^k = 1_G} &
  H is normal & K is normal
  &
  (for x be Element of G holds
  ex a,b be Element of G st a in H & b in K & x = a*b)
  &
  (the carrier of H) /\ (the carrier of K) = {1_G};
```

3 Results

Nulla mi mi, venenatis sed ipsum varius, Table 1 volutpat euismod diam. Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. Fusce fringilla erat porttitor lectus cursus, vel sagittis arcu lobortis. Aliquam in enim semper, aliquam massa id, cursus neque. Praesent faucibus semper libero.

表 1. Table caption Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam.

Heading1		**Heading2**	
$cell1row1$	cell2 row 1	cell3 row 1	cell4 row 1
$cell1row2$	cell2 row 2	cell3 row 2	cell4 row 2
$cell1row3$	cell2 row 3	cell3 row 3	cell4 row 3

Table notes Phasellus venenatis, tortor nec vestibulum mattis, massa tortor interdum felis, nec pellentesque metus tortor nec nisl. Ut ornare mauris tellus, vel dapibus arcu suscipit sed.

3.1 LOREM and IPSUM Nunc blandit a tortor.

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3.2 Sed ac quam id nisi malesuada congue.

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3.3 Subsection 1

Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam. Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. Fusce fringilla erat porttitor lectus cursus, vel sagittis arcu lobortis. Aliquam in enim semper, aliquam massa id, cursus neque. Praesent faucibus semper libero.

3.4 Subsection 2

3rd Level Heading. Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam. Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. Fusce fringilla erat porttitor lectus cursus, vel sagittis arcu lobortis. Aliquam in enim semper, aliquam massa id, cursus neque. Praesent faucibus semper libero.

4 Discussion

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4.2 LOREM and IPSUM Nunc blandit a tortor.

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5 Conclusions

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Acknowledgments

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Mizar article information

Mizar Mathematical Library (MML)

GROUP_4 Lattice of Subgroups of a Group [4]

```
notations TARSKI, XBOOLE_0, ZFMISC_1, SUBSET_1, XCMPLX_0, FINSOP_1, ORDINAL1,
      NUMBERS, INT_1, SETWISEO, SETFAM_1, FUNCT_1, PARTFUN1, FUNCT_2, FINSEQ_1,
      FINSEQ_2, FINSEQ_3, FINSEQ_4, BINOP_1, STRUCT_0, ALGSTR_0, GROUP_2,
      GROUP_3, LATTICES, GROUP_1, DOMAIN_1, XXREAL_0, NAT_1, INT_2;
```

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GROUP_7 The Product of the Families of the Groups [5]

```
notations TARSKI, XBOOLE_0, ENUMSET1, ZFMISC_1, SUBSET_1, RELAT_1, FUNCT_1,
      ORDINAL1, NAT_1, FINSEQ_1, RELSET_1, PARTFUN1, FUNCT_2, FUNCT_4,
      FINSET_1, BINOP_1, REALSET1, XXREAL_0, PBOOLE, FUNCOP_1, STRUCT_0,
      ALGSTR_0, MONOID_0, GROUP_1, GROUP_2, GROUP_6, CARD_3, PRALG_1;
```

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Works in Progress

GROUP_17 Isomorphisms of Direct Products of Finite Commutative Groups
by Hiroyuki Okazaki, Hiroshi Yamazaki and Yasunari Shidama

```
notations TARSKI, XBOOLE_0, ZFMISC_1, SUBSET_1, RELAT_1, FUNCT_1, ORDINAL1,
      RELSET_1, PARTFUN1, FUNCT_2, DOMAIN_1, FUNCOP_1, FUNCT_4, FINSET_1,
      CARD_1, PBOOLE, CARD_3, NUMBERS, XCMPLX_0, XXREAL_0, XREAL_0, NAT_1,
      INT_1, INT_2, BINOP_1, FINSEQ_1, NEWTON, PRE_POLY, NAT_3, STRUCT_0,
      ALGSTR_0, GROUP_1, GROUP_2, GROUP_3, GROUP_4, GROUP_6, PRALG_1, GROUP_7,
      INT_7;
```

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Listing 2. GROUP_17 - abstract

```
:: Isomorphisms of Direct Products of Finite Commutative Groups
:: by Hiroyuki Okazaki , Hiroshi Yamazaki and Yasunari Shidama

environ

 vocabularies FINSEQ_1, FUNCT_1, RELAT_1, RLVECT_2, CARD_3, TARSKI, BINOP_1,
      GROUP_1, XXREAL_0, GROUP_2, CARD_1, FUNCT_4, GROUP_6, GROUP_7, FUNCOP_1,
      ALGSTR_0, PARTFUN1, FUNCT_2, SUBSET_1, XBOOLE_0, STRUCT_0, NAT_1,
      ORDINAL4, PRE_TOPC, ARYTM_1, ARYTM_3, FINSET_1, INT_2, ZFMISC_1, PBOOLE,
      NEWTON, INT_1, NAT_3, REAL_1, PRE_POLY, XCMPLX_0, UPROOTS, INT_7;
 notations TARSKI, XBOOLE_0, ZFMISC_1, SUBSET_1, RELAT_1, FUNCT_1, ORDINAL1,
      RELSET_1, PARTFUN1, FUNCT_2, DOMAIN_1, FUNCOP_1, FUNCT_4, FINSET_1,
      CARD_1, PBOOLE, CARD_3, NUMBERS, XCMPLX_0, XXREAL_0, XREAL_0, NAT_1,
      INT_1, INT_2, BINOP_1, FINSEQ_1, NEWTON, PRE_POLY, NAT_3, STRUCT_0,
      ALGSTR_0, GROUP_1, GROUP_2, GROUP_3, GROUP_4, GROUP_6, PRALG_1, GROUP_7,
      INT_7;
 constructors BINOP_1, REALSET1, GROUP_6, MONOID_0, PRALG_1, GROUP_4, CARD_2,
      GROUP_7, RELSET_1, WELLORD2, NAT_D, INT_7, RECDEF_1, NAT_3, FINSOP_1;
```

```
registrations XBOOLE_0, XREAL_0, STRUCT_0, GROUP_2, MONOID_0, FUNCT_2, CARD_1,
    CARD_3, GROUP_7, GROUP_3, RELSET_1, FINSEQ_1, INT_1, AOFA_000, GR_CY_1,
    FINSET_1, NAT_3, RELAT_1, FUNCT_1, MEMBERED, FUNCOP_1, NEWTON, VALUED_0,
    PRE_POLY, PBOOLE, INT_7, GROUP_6, ORDINAL1;
requirements NUMERALS, SUBSET, ARITHM, BOOLE;


begin :: Preliminaries

theorem :: GROUP_17:1
  for A,B,A1,B1 be set st A misses B
  & A1 c= A & B1 c= B & A1 \/ B1 = A \/ B holds
  A1 = A & B1 = B;

theorem :: GROUP_17:2
  for H,K be non empty finite set holds
  card product (<* H, K *>) = card(H)*card(K);

theorem :: GROUP_17:3
  for ps,pt,f be bag of SetPrimes,
  q being Nat
  st (support ps) misses (support pt) & f = ps + pt & q in (support ps) holds
  ps.q = f.q;

theorem :: GROUP_17:4
  for ps,pt,f be bag of SetPrimes,
  q being Nat
  st (support ps) misses (support pt) & f = ps + pt & q in (support pt) holds
  pt.q = f.q;

theorem :: GROUP_17:5
  for h be non zero Nat, q being Prime
  st not q,h are_coprime holds
  q divides h;

theorem :: GROUP_17:6
  for h,s be non zero Nat
  st for q being Prime st q in support (prime_factorization s)
  holds not q,h are_coprime holds
  support (prime_factorization s) c= support (prime_factorization h);

theorem :: GROUP_17:7
  for h,k,s,t be non zero Nat
  st h,k are_coprime & s * t = h * k
  & (for q being Prime st q in support prime_factorization s
  holds not q,h are_coprime)
  & (for q being Prime st q in support prime_factorization t
  holds not q,k are_coprime)
  holds
  s = h & t = k;

definition
  let G be non empty multMagma,
  I be finite set,
  b be (the carrier of G)-valued total I -defined Function;
  func Product b -> Element of G means
:: GROUP_17:def 1
  ex f being FinSequence of G st it = Product f & f = b*canFS(I);
end;

theorem :: GROUP_17:8
  for G being commutative Group,
  A,B being non empty finite set,
  FA be (the carrier of G)-valued total A -defined Function,
  FB be (the carrier of G)-valued total B -defined Function,
  FAB be (the carrier of G)-valued total A \/ B -defined Function
  st A misses B & FAB = FA +* FB holds
  Product (FAB) = (Product FA) * (Product FB);
```

```
theorem :: GROUP_17:9
  for G being non empty multMagma,
  q be set,
  z be Element of G,
  f be (the carrier of G)-valued total {q}-defined Function
  st f = q .--> z
  holds Product f = z;

begin :: Direct Product of Finite Commutative Groups

theorem :: GROUP_17:10
  for X,Y being non empty multMagma holds
  the carrier of product <*X,Y*>
  = product <* the carrier of X,the carrier of Y *>;

theorem :: GROUP_17:11
  for G being Group, A,B being normal Subgroup of G st
  (the carrier of A) /\ (the carrier of B) = {1_G} holds
  for a,b be Element of G st a in A & b in B holds a*b = b*a;

theorem :: GROUP_17:12
  for G being Group, A,B being normal Subgroup of G st
  (for x be Element of G holds
  ex a,b be Element of G st a in A & b in B & x = a*b)
  & (the carrier of A) /\ (the carrier of B) = {1_G} holds
  ex h being Homomorphism of product <*A,B*>,G st h is bijective
  & for a,b be Element of G st a in A & b in B
  holds h.(<*a,b*>) = a*b;

theorem :: GROUP_17:13
  for G being finite commutative Group,
  m be Nat,
  A be Subset of G
  st A ={x where x is Element of G: x|^m = 1_G}
  holds
  A <> {}
  &
  (for g1,g2 be Element of G
  st g1 in A & g2 in A holds g1 * g2 in A) &
  for g be Element of G st g in A holds g" in A;

theorem :: GROUP_17:14
  for G being finite commutative Group,
  m be Nat,
  A be Subset of G
  st A ={x where x is Element of G: x|^m = 1_G} holds
  ex H being strict finite Subgroup of G
  st the carrier of H = A & H is commutative normal;

theorem :: GROUP_17:15
  for G being finite commutative Group,
  m be Nat,
  H being finite Subgroup of G
  st the carrier of H = {x where x is Element of G: x|^m = 1_G} holds
  for q being Prime st q in support prime_factorization card H
  holds not q,m are_coprime;

theorem :: GROUP_17:16
  for G being finite commutative Group,
  h,k be Nat
  st card G = h*k & h,k are_coprime holds
  ex H,K being strict finite Subgroup of G st
  the carrier of H = {x where x is Element of G: x|^h = 1_G} &
  the carrier of K = {x where x is Element of G: x|^k = 1_G} &
  H is normal & K is normal
  &
  (for x be Element of G holds
  ex a,b be Element of G st a in H & b in K & x = a*b)
  &
```

```
  (the carrier of H) /\ (the carrier of K) = {1_G};

theorem :: GROUP_17:17
  for H,K be finite Group holds
  card product (<* H, K *>) = card(H)*card(K);

theorem :: GROUP_17:18
  for G being finite commutative Group,
  h,k be non zero Nat
  st card G = h*k & h,k are_coprime
  ex H,K being strict finite Subgroup of G st
  card H = h & card K = k &
  (the carrier of H) /\ (the carrier of K) = {1_G} &
  ex F being Homomorphism of product <*H,K*>,G
  st F is bijective
  & for a,b be Element of G st a in H & b in K
  holds F.(<*a,b*>) = a*b;

begin :: Finite Direct Products of Finite Commutative Groups

theorem :: GROUP_17:19
  for G be Group,
  q be set,
  F be associative Group-like multMagma-Family of {q},
  f being Function of G,product F st F = q .--> G &
  for x being Element of G holds f . x = q .--> x holds
  f is Homomorphism of G,(product F);

theorem :: GROUP_17:20
  for G be Group,
  q be set,
  F be associative Group-like multMagma-Family of {q},
  f being Function of G,product F st F = q .--> G &
  for x being Element of G holds f . x = q .--> x holds
  f is bijective;

theorem :: GROUP_17:21
  for q be set,
  F be associative Group-like multMagma-Family of {q},
  G be Group st F = q .--> G holds
  ex I be Homomorphism of G,product F st
  I is bijective &
  for x being Element of G holds I . x = q .--> x;

theorem :: GROUP_17:22
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  k be Element of K,
  g be Function st
  g in the carrier of product F0 &
  not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) holds
  g +* (q .--> k) in the carrier of product F;

theorem :: GROUP_17:23
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  G0 be Function of H,product F0 st
  G0 is Homomorphism of H,product F0
  & G0 is bijective & not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) holds
  for G be Function of product <*H,K*>,(product F) st
  for h be Element of H,k be Element of K
  holds ex g be Function
  st g=G0.h & G.(<*h,k*>) = g +* (q .--> k) holds
```

```
  G is Homomorphism of product <*H,K*>,product F;

theorem :: GROUP_17:24
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  G0 be Function of H, product F0 st
  G0 is Homomorphism of H, product F0
  & G0 is bijective
  & not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) holds
  for G be Function of product <*H,K*>, product F st
  for h be Element of H,k be Element of K
  holds ex g be Function
  st g=G0.h & G.(<*h,k*>) = g +* (q .--> k)
  holds G is bijective;

theorem :: GROUP_17:25
  for q be set,
  F be multMagma-Family of {q},
  G be non empty multMagma st
  F = q .--> G holds
  for y be (the carrier of G)-valued total {q} -defined Function holds
  y in the carrier of product F & y.q in the carrier of G &
  y= q .--> y.q;

theorem :: GROUP_17:26
  for q be set,
  F be associative Group-like multMagma-Family of {q},
  G be Group st F = q .--> G holds
  ex HFG be Homomorphism of product F,G st
  HFG is bijective &
  for x be (the carrier of G)-valued total {q} -defined Function
  holds HFG.x = Product x;

theorem :: GROUP_17:27
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  G0 be Homomorphism of H,(product F0) st
  not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) & G0 is bijective
  ex G be Homomorphism of product <*H,K*>,(product F) st
  G is bijective &
  for h be Element of H,k be Element of K
  ex g be Function st g=G0.h & G.(<*h,k*>) = g +* (q .--> k);

theorem :: GROUP_17:28
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  G0 be Homomorphism of product F0, H st not q in I0 &
  I = I0 \/ {q} & F = F0 +* (q .--> K) & G0 is bijective holds
  ex G be Homomorphism of product F, product <*H,K*> st G is bijective &
  for x0 be Function,
  k be Element of K,
  h be Element of H
  st h = G0.x0 & x0 in product F0 holds
  G.(x0 +* (q .-->k)) = <* h, k *>;

theorem :: GROUP_17:29
  for I be non empty finite set,
  F be associative Group-like multMagma-Family of I,
  x be total I -defined Function
  st for p be Element of I holds x.p in F.p
```

```
holds x in the carrier of product F;

theorem :: GROUP_17:30
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  K be Group,
  q be Element of I,
  x be Element of product F st
  not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) holds
  ex x0 be total I0 -defined Function,
  k be Element of K st x0 in product F0
  & x = x0 +* (q .--> k) & for p be Element of I0 holds x0.p in F0.p;

theorem :: GROUP_17:31
  for G be Group,
  H be Subgroup of G,
  f being FinSequence of G,
  g being FinSequence of H
  st f=g
  holds Product f = Product g;

theorem :: GROUP_17:32
  for I be non empty finite set,
  G be Group,
  H be Subgroup of G,
  x be (the carrier of G)-valued total I -defined Function,
  x0 be (the carrier of H)-valued total I -defined Function
  st x=x0
  holds Product x = Product x0;

theorem :: GROUP_17:33
  for G being commutative Group,
  I0,I be non empty finite set,
  q be Element of I,
  x be (the carrier of G)-valued total I -defined Function,
  x0 be (the carrier of G)-valued total I0 -defined Function,
  k be Element of G st
  not q in I0 & I = I0 \/ {q} & x = x0 +* (q .--> k)
  holds
  Product x = (Product x0)*k;

theorem :: GROUP_17:34
  for G being strict finite commutative Group
  st card G > 1 holds
  ex I be non empty finite set,
  F be associative Group-like commutative multMagma-Family of I,
  HFG be Homomorphism of product F,G st
  I = support (prime_factorization card G)
  & (for p be Element of I holds F.p is strict Subgroup of G &
  card (F.p) = (prime_factorization card G).p) &
  (for p,q be Element of I st p <> q holds
  (the carrier of (F.p)) /\ (the carrier of (F.q)) = {1_G}) &
  HFG is bijective &
  for x be (the carrier of G)-valued total I -defined Function
  st for p be Element of I holds x.p in F.p
  holds x in product F & HFG.x = Product x;

theorem :: GROUP_17:35
  for G being strict finite commutative Group st card G > 1 holds
  ex I be non empty finite set,
  F be associative Group-like commutative multMagma-Family of I st
  I = support (prime_factorization card G)
  & (for p be Element of I holds F.p is strict Subgroup of G &
  card (F.p) = (prime_factorization card G).p) &
  (for p,q be Element of I st p <> q holds
  (the carrier of (F.p)) /\ (the carrier of (F.q)) = {1_G})
  &
  (for y be Element of G
```

ex x **be** (**the** carrier **of** G)−valued total I −defined Function
st (**for** p **be** Element **of** I **holds** x.p in F.p) **&** y = Product x)
&
for x1,x2 **be** (**the** carrier **of** G)−valued total I −defined Function **st**
(**for** p **be** Element **of** I **holds** x1.p in F.p) **&**
(**for** p **be** Element **of** I **holds** x2.p in F.p) **&**
Product x1 = Product x2 **holds** x1=x2;