Regular Paper

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Abstract

アブストラクトをここに書く. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Curabitur eget porta erat. Morbi consectetur est vel gravida pretium. Suspendisse ut dui eu ante cursus gravida non sed sem. Nullam sapien tellus, commodo id velit id, eleifend volutpat quam. Phasellus mauris velit, dapibus finibus elementum vel, pulvinar non tellus. Nunc pellentesque pretium diam, quis maximus dolor faucibus id.

1 Introduction

イントロダクションをここに書く. Lorem ipsum dolor sit [1] amet, consectetur adipiscing elit. Curabitur eget porta erat. Morbi consectetur est vel gravida pretium. Suspendisse ut dui eu ante cursus gravida non sed sem. Nullam Eq. (1) sapien tellus, commodo id velit id, eleifend volutpat quam. Phasellus mauris velit, dapibus finibus elementum vel, pulvinar non tellus. Nunc pellentesque pretium diam, quis maximus dolor faucibus id. [2] Nunc convallis sodales ante, ut ullamcorper est egestas vitae. Nam sit amet enim ultrices, ultrices elit pulvinar, volutpat risus.

$$D_{coll} = \frac{D_f + \frac{[S]^2}{K_D S_T} D_S}{1 + \frac{[S]^2}{K_D S_T}}, D_{sm} = \frac{D_f + \frac{[S]}{K_D} D_S}{1 + \frac{[S]}{K_D}} \quad (1)$$

2 Methods

2.1 Etiam eget sapien nibh.

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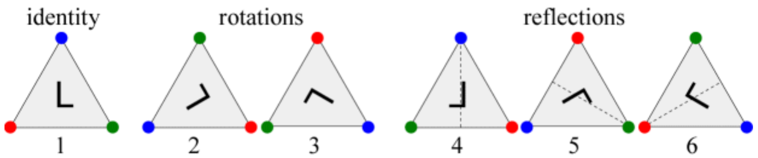

Fig.1. Figure Title first bold sentence Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam. Figure Caption Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. A: Lorem ipsum dolor sit amet. B: Consectetur adipiscing elit.

2.2 Formalizations

Nulla mi mi, Proin rutrum vel massa non gravida. Quisque tempor sem et [41] dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. See Listing 1.

Listing 1. GROUP_17 - Th.16

```
theorem :: GROUP 17:16
   for G being finite commutative Group,
   h,k be Nat
   st card G = h*k & h,k are coprime holds
```

```
ex H,K being strict finite Subgroup of G st
the carrier of H = {x where x is Element of G: x|^h = 1 G} &        :: 注釈を入れてもよい
the carrier of K = {x where x is Element of G: x|^k = 1 G} &
H is normal & K is normal
&
(for x be Element of G holds
ex a,b be Element of G st a in H & b in K & x = a * b)
&
(the carrier of H) /\ (the carrier of K) = {1_G};
```

3 Results

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Table 1. Table caption Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam.

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cellrow	cellrow	cellrow	cellrow	cellrow	cellrow
cellrow	cellrow	cellrow	cellrow	cellrow	cellrow
cellrow	cellrow	cellrow	cellrow	cellrow	cellrow

Table notes Phasellus venenatis, tortor nec vestibulum mattis, massa tortor interdum felis, nec pellentesque metus tortor nec nisl. Ut ornare mauris tellus, vel dapibus arcu suscipit sed.

3.1 LOREM and IPSUM Nunc blandit a tortor.

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3.2 Sed ac quam id nisi malesuada congue. 74

Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam. Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. Fusce fringilla erat porttitor lectus cursus, vel sagittis arcu lobortis. Aliquam in enim semper, aliquam massa id, cursus neque. Praesent faucibus semper libero.

3.3 Subsection 1

Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam. Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. Fusce fringilla erat porttitor lectus cursus, vel sagittis arcu lobortis. Aliquam in enim semper, aliquam massa id, cursus neque. Praesent faucibus semper libero.

3.4 Subsection 2

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4 Discussion

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4.1 LOREM and IPSUM Nunc blandit a tortor.

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4.2 LOREM and IPSUM Nunc blandit a tortor.

Nulla mi mi, venenatis sed ipsum varius, volutpat euismod diam. Proin rutrum vel massa non gravida. Quisque tempor sem et dignissim rutrum. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Morbi at justo vitae nulla elementum commodo eu id massa. In vitae diam ac augue semper tincidunt eu ut eros. Fusce fringilla erat porttitor lectus cursus, vel sagittis arcu lobortis. Aliquam in enim semper, aliquam massa id, cursus neque. Praesent faucibus semper libero.

5 Conclusions

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Acknowledgments

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congue. Sed interdum aliquet augue, at pellentesque quam rhoncus vitae. See MML reference [3], [4] and [5].

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Mizar article information

Works in Progress

GROUP_17 Isomorphisms of Direct Products of Finite Commutative Groups

by Hiroyuki Okazaki, Hiroshi Yamazaki and Yasunari Shidama

```
notations TARSKI, XBOOLE_0, ZFMISC_1, SUBSET_1, RELAT_1, FUNCT_1, ORDINAL1,
    RELSET_1,PARTFUN1, FUNCT_2, DOMAIN_1, FUNCOP_1, FUNCT_4, FINSET_1,
    CARD_1, PBOOLE, CARD_3, NUMBERS, XCMPLX_0, XXREAL_0, XREAL_0, NAT_1
    INT_1, INT_2, BINOP_1, FINSEQ_1, NEWTON, PRE_POLY, NAT_3, STRUCT_0,
    ALGSTR_0, GROUP_1, GROUP_2, GROUP_3, GROUP_4, GROUP_6, PRALG_1, GROUP_7,
    INT_7;
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Listing 2. GROUP_17 - abstract

```
:: Isomorphisms of Direct Products of Finite Commutative Groups
::  by Hiroyuki Okazaki , Hiroshi Yamazaki and Yasunari Shidama

environ

 vocabularies FINSEQ_1, FUNCT_1, RELAT_1, RLVECT_2, CARD_3, TARSKI, BINOP_1,
      GROUP_1, XXREAL_0, GROUP_2, CARD_1, FUNCT_4, GROUP_6, GROUP_7, FUNCOP_1,
      ALGSTR_0, PARTFUN1, FUNCT_2, SUBSET_1, XBOOLE_0, STRUCT_0, NAT_1,
      ORDINAL4, PRE_TOPC, ARYTM_1, ARYTM_3, FINSET_1, INT_2, ZFMISC_1, PBOOLE,
      NEWTON, INT_1, NAT_3, REAL_1, PRE_POLY, XCMPLX_0, UPROOTS, INT_7;
 notations TARSKI, XBOOLE_0, ZFMISC_1, SUBSET_1, RELAT_1, FUNCT_1, ORDINAL1,
      RELSET_1, PARTFUN1, FUNCT_2, DOMAIN_1, FUNCOP_1, FUNCT_4, FINSET_1,
      CARD_1, PBOOLE, CARD_3, NUMBERS, XCMPLX_0, XXREAL_0, XREAL_0, NAT_1,
      INT_1, INT_2, BINOP_1, FINSEQ_1, NEWTON, PRE_POLY, NAT_3, STRUCT_0,
      ALGSTR_0, GROUP_1, GROUP_2, GROUP_3, GROUP_4, GROUP_6, PRALG_1, GROUP_7,
      INT_7;
 constructors BINOP_1, REALSET1, GROUP_6, MONOID_0, PRALG_1, GROUP_4, CARD_2,
      GROUP_7, RELSET_1, WELLORD2, NAT_D, INT_7, RECDEF_1, NAT_3, FINSOP_1;
 registrations XBOOLE_0, XREAL_0, STRUCT_0, GROUP_2, MONOID_0, FUNCT_2, CARD_1,
      CARD_3, GROUP_7, GROUP_3, RELSET_1, FINSEQ_1, INT_1, AOFA_000, GR_CY_1,
      FINSET_1, NAT_3, RELAT_1, FUNCT_1, MEMBERED, FUNCOP_1, NEWTON, VALUED_0,
      PRE_POLY, PBOOLE, INT_7, GROUP_6, ORDINAL1;
 requirements NUMERALS, SUBSET, ARITHM, BOOLE;

begin :: Preliminaries

theorem :: GROUP_17:1
  for A,B,A1,B1 be set st A misses B
  & A1 c= A & B1 c= B & A1 \/ B1 = A \/ B holds
  A1 = A & B1 = B;

theorem :: GROUP_17:2
  for H,K be non empty finite set holds
  card product (<* H, K *>) = card(H)*card(K);

theorem :: GROUP_17:3
  for ps,pt,f be bag of SetPrimes,
  q being Nat
  st (support ps) misses (support pt) & f = ps + pt & q in (support ps) holds
  ps.q = f.q;

theorem :: GROUP_17:4
  for ps,pt,f be bag of SetPrimes,
  q being Nat
  st (support ps) misses (support pt) & f = ps + pt & q in (support pt) holds
  pt.q = f.q;

theorem :: GROUP_17:5
  for h be non zero Nat, q being Prime
  st not q,h are_coprime holds
  q divides h;

theorem :: GROUP_17:6
  for h,s be non zero Nat
  st for q being Prime st q in support (prime_factorization s)
  holds not q,h are_coprime holds
  support (prime_factorization s) c= support (prime_factorization h);

theorem :: GROUP_17:7
  for h,k,s,t be non zero Nat
```

```
  st h,k are_coprime & s * t = h * k
  & (for q being Prime st q in support prime_factorization s
  holds not q,h are_coprime)
  & (for q being Prime st q in support prime_factorization t
  holds not q,k are_coprime)
  holds
  s = h & t = k;

definition
  let G be non empty multMagma,
  I be finite set,
  b be (the carrier of G)-valued total I -defined Function;
  func Product b -> Element of G means
:: GROUP_17:def 1
  ex f being FinSequence of G st it = Product f & f = b*canFS(I);
end;

theorem :: GROUP_17:8
  for G being commutative Group,
  A,B being non empty finite set,
  FA be (the carrier of G)-valued total A -defined Function,
  FB be (the carrier of G)-valued total B -defined Function,
  FAB be (the carrier of G)-valued total A \/ B -defined Function
  st A misses B & FAB = FA +* FB holds
  Product (FAB) = (Product FA) * (Product FB);

theorem :: GROUP_17:9
  for G being non empty multMagma,
  q be set,
  z be Element of G,
  f be (the carrier of G)-valued total {q}-defined Function
  st f = q .--> z
  holds Product f = z;

begin :: Direct Product of Finite Commutative Groups

theorem :: GROUP_17:10
  for X,Y being non empty multMagma holds
  the carrier of product <*X,Y*>
  = product <* the carrier of X,the carrier of Y *>;

theorem :: GROUP_17:11
  for G being Group, A,B being normal Subgroup of G st
  (the carrier of A) /\ (the carrier of B) = {1_G} holds
  for a,b be Element of G st a in A & b in B holds a*b = b*a;

theorem :: GROUP_17:12
  for G being Group, A,B being normal Subgroup of G st
  (for x be Element of G holds
  ex a,b be Element of G st a in A & b in B & x = a*b)
  & (the carrier of A) /\ (the carrier of B) = {1_G} holds
  ex h being Homomorphism of product <*A,B*>,G st h is bijective
  & for a,b be Element of G st a in A & b in B
  holds h.(<*a,b*>) = a*b;

theorem :: GROUP_17:13
  for G being finite commutative Group,
  m be Nat,
  A be Subset of G
  st A ={x where x is Element of G: x|^m = 1_G}
  holds
  A <> {}
  &
  (for g1,g2 be Element of G
  st g1 in A & g2 in A holds g1 * g2 in A) &
  for g be Element of G st g in A holds g" in A;

theorem :: GROUP_17:14
  for G being finite commutative Group,
  m be Nat,
```

```
  A be Subset of G
  st A ={x where x is Element of G: x|^m = 1_G} holds
  ex H being strict finite Subgroup of G
  st the carrier of H = A & H is commutative normal;

theorem :: GROUP_17:15
  for G being finite commutative Group,
  m be Nat,
  H being finite Subgroup of G
  st the carrier of H = {x where x is Element of G: x|^m = 1_G} holds
  for q being Prime st q in support prime_factorization card H
  holds not q,m are_coprime;

theorem :: GROUP_17:16
  for G being finite commutative Group,
  h,k be Nat
  st card G = h*k & h,k are_coprime holds
  ex H,K being strict finite Subgroup of G st
  the carrier of H = {x where x is Element of G: x|^h = 1_G} &
  the carrier of K = {x where x is Element of G: x|^k = 1_G} &
  H is normal & K is normal
  &
  (for x be Element of G holds
  ex a,b be Element of G st a in H & b in K & x = a*b)
  &
  (the carrier of H) /\ (the carrier of K) = {1_G};

theorem :: GROUP_17:17
  for H,K be finite Group holds
  card product (<* H, K *>) = card(H)*card(K);

theorem :: GROUP_17:18
  for G being finite commutative Group,
  h,k be non zero Nat
  st card G = h*k & h,k are_coprime
  ex H,K being strict finite Subgroup of G st
  card H = h & card K = k &
  (the carrier of H) /\ (the carrier of K) = {1_G} &
  ex F being Homomorphism of product <*H,K*>,G
  st F is bijective
  & for a,b be Element of G st a in H & b in K
  holds F.(<*a,b*>) = a*b;

begin :: Finite Direct Products of Finite Commutative Groups

theorem :: GROUP_17:19
  for G be Group,
  q be set,
  F be associative Group-like multMagma-Family of {q},
  f being Function of G,product F st F = q .--> G &
  for x being Element of G holds f . x = q .--> x holds
  f is Homomorphism of G,(product F);

theorem :: GROUP_17:20
  for G be Group,
  q be set,
  F be associative Group-like multMagma-Family of {q},
  f being Function of G,product F st F = q .--> G &
  for x being Element of G holds f . x = q .--> x holds
  f is bijective;

theorem :: GROUP_17:21
  for q be set,
  F be associative Group-like multMagma-Family of {q},
  G be Group st F = q .--> G holds
  ex I be Homomorphism of G,product F st
  I is bijective &
  for x being Element of G holds I . x = q .--> x;

theorem :: GROUP_17:22
```

```
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  k be Element of K,
  g be Function st
  g in the carrier of product F0 &
  not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) holds
  g +* (q .--> k) in the carrier of product F;

theorem :: GROUP_17:23
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  G0 be Function of H,product F0 st
  G0 is Homomorphism of H,product F0
  & G0 is bijective & not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) holds
  for G be Function of product <*H,K*>,(product F) st
  for h be Element of H,k be Element of K
  holds ex g be Function
  st g=G0.h & G.(<*h,k*>) = g +* (q .--> k) holds
  G is Homomorphism of product <*H,K*>,product F;

theorem :: GROUP_17:24
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  G0 be Function of H, product F0 st
  G0 is Homomorphism of H, product F0
  & G0 is bijective
  & not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) holds
  for G be Function of product <*H,K*>, product F st
  for h be Element of H,k be Element of K
  holds ex g be Function
  st g=G0.h & G.(<*h,k*>) = g +* (q .--> k)
  holds G is bijective;

theorem :: GROUP_17:25
  for q be set,
  F be multMagma-Family of {q},
  G be non empty multMagma st
  F = q .--> G holds
  for y be (the carrier of G)-valued total {q} -defined Function holds
  y in the carrier of product F & y.q in the carrier of G &
  y= q .--> y.q;

theorem :: GROUP_17:26
  for q be set,
  F be associative Group-like multMagma-Family of {q},
  G be Group st F = q .--> G holds
  ex HFG be Homomorphism of product F,G st
  HFG is bijective &
  for x be (the carrier of G)-valued total {q} -defined Function
  holds HFG.x = Product x;

theorem :: GROUP_17:27
  for I0,I be non empty finite set,
  F0 be associative Group-like multMagma-Family of I0,
  F be associative Group-like multMagma-Family of I,
  H,K be Group,
  q be Element of I,
  G0 be Homomorphism of H,(product F0) st
  not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) & G0 is bijective
  ex G be Homomorphism of product <*H,K*>,(product F) st
  G is bijective &
```

```
    for h be Element of H,k be Element of K
    ex g be Function st g=G0.h & G.(<*h,k*>) = g +* (q .--> k);

  theorem :: GROUP_17:28
    for I0,I be non empty finite set,
    F0 be associative Group-like multMagma-Family of I0,
    F be associative Group-like multMagma-Family of I,
    H,K be Group,
    q be Element of I,
    G0 be Homomorphism of product F0, H st not q in I0 &
    I = I0 \/ {q} & F = F0 +* (q .--> K) & G0 is bijective holds
    ex G be Homomorphism of product F, product <*H,K*> st G is bijective &
    for x0 be Function,
    k be Element of K,
    h be Element of H
    st h = G0.x0 & x0 in product F0 holds
    G.(x0 +* (q .-->k)) = <* h, k *>;

  theorem :: GROUP_17:29
    for I be non empty finite set,
    F be associative Group-like multMagma-Family of I,
    x be total I -defined Function
    st for p be Element of I holds x.p in F.p
    holds x in the carrier of product F;

  theorem :: GROUP_17:30
    for I0,I be non empty finite set,
    F0 be associative Group-like multMagma-Family of I0,
    F be associative Group-like multMagma-Family of I,
    K be Group,
    q be Element of I,
    x be Element of product F st
    not q in I0 & I = I0 \/ {q} & F = F0 +* (q .--> K) holds
    ex x0 be total I0 -defined Function,
    k be Element of K st x0 in product F0
    & x = x0 +* (q .--> k) & for p be Element of I0 holds x0.p in F0.p;

  theorem :: GROUP_17:31
    for G be Group,
    H be Subgroup of G,
    f being FinSequence of G,
    g being FinSequence of H
    st f=g
    holds Product f = Product g;

  theorem :: GROUP_17:32
    for I be non empty finite set,
    G be Group,
    H be Subgroup of G,
    x be (the carrier of G)-valued total I -defined Function,
    x0 be (the carrier of H)-valued total I -defined Function
    st x=x0
    holds Product x = Product x0;

  theorem :: GROUP_17:33
    for G being commutative Group,
    I0,I be non empty finite set,
    q be Element of I,
    x be (the carrier of G)-valued total I -defined Function,
    x0 be (the carrier of G)-valued total I0 -defined Function,
    k be Element of G st
    not q in I0 & I = I0 \/ {q} & x = x0 +* (q .--> k)
    holds
    Product x = (Product x0)*k;

  theorem :: GROUP_17:34
    for G being strict finite commutative Group
    st card G > 1 holds
    ex I be non empty finite set,
    F be associative Group-like commutative multMagma-Family of I,
```

```
  HFG be Homomorphism of product F,G st
  I = support (prime_factorization card G)
  & (for p be Element of I holds F.p is strict Subgroup of G &
  card (F.p) = (prime_factorization card G).p) &
  (for p,q be Element of I st p <> q holds
  (the carrier of (F.p)) /\ (the carrier of (F.q)) = {1_G}) &
  HFG is bijective &
  for x be (the carrier of G)-valued total I -defined Function
  st for p be Element of I holds x.p in F.p
  holds x in product F & HFG.x = Product x;

theorem :: GROUP_17:35
  for G being strict finite commutative Group st card G > 1 holds
  ex I be non empty finite set,
  F be associative Group-like commutative multMagma-Family of I st
  I = support (prime_factorization card G)
  & (for p be Element of I holds F.p is strict Subgroup of G &
  card (F.p) = (prime_factorization card G).p) &
  (for p,q be Element of I st p <> q holds
  (the carrier of (F.p)) /\ (the carrier of (F.q)) = {1_G})
  &
  (for y be Element of G
    ex x be (the carrier of G)-valued total I -defined Function
  st (for p be Element of I holds x.p in F.p) & y = Product x)
  &
  for x1,x2 be (the carrier of G)-valued total I -defined Function st
  (for p be Element of I holds x1.p in F.p) &
  (for p be Element of I holds x2.p in F.p) &
  Product x1 = Product x2 holds x1=x2;
```