Article

On n-dimensional Real Spaces and n-dimensional Complex Spaces I

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Abstract

A stationary object and a moving object are different entities. Light, heat, electromagnetic waves, and the moving stars of the universe have similar characteristics. In conventional physics, the state of an object is described by a combination of the coordinates q of position and the coordinates p of momentum. In fact, this description method is suitable for describing the state of many other physical quantities. We develop a basic theory that expresses the duality of such existence.

静止している物体と，運動している物体とでは異なる存在である．光も熱も電磁波も宇宙の運動する宇宙の星々も同様な特徴を持つ．従来の物理学では，位置の座標 q と運動量の座標 p の座標の組み合わせで物体の状態を記述した．実はこの記述法は，他の多くの物理量の状態記述に適しているのである．そのような存在の二面性を表現する基礎理論を展開する．

Mizar article information

Works in Progress

MOMENTM1 On n-dimensional Real Spaces and n-dimensional Complex Spaces I
by Yatsuka Nakamura

Listing 1. MOMENTM1 - abstract (momentm1.abs)

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::
:: On n-dimensional Real Spaces and n-dimensional Complex Spaces I
:: 19. Nov. 2020
:: Yatsuka Nakamura

environ

 vocabularies TARSKI, NUMBERS, NAT_1, FINSEQ_2, FINSEQ_1, SUBSET_1, SQUARE_1,
      VALUED_0, CARD_1, XXREAL_0, XCMPLX_0, FUNCT_1, FUNCT_2, FUNCT_7,
      XBOOLE_0, RVSUM_1, COMPLEX1, RELAT_1, REAL_1, ARYTM_1, MOMENTM1;
 notations TARSKI, XBOOLE_0, VALUED_0, FUNCT_1, FUNCT_2, ORDINAL1, CARD_1,
      NUMBERS, XCMPLX_0, XREAL_0, COMPLEX1, FINSEQ_1, FINSEQ_2, DOMAIN_1,
      RELAT_1, XXREAL_0, SUBSET_1, EUCLID;
 constructors COMPLEX1, FINSEQOP, TOPMETR, EUCLID;
 registrations RELSET_1, NUMBERS, XREAL_0, FINSEQ_2, RVSUM_1, ORDINAL1,
      COMPLEX1, XCMPLX_0;
 requirements NUMERALS, SUBSET, BOOLE;
 definitions TARSKI;
 equalities EUCLID, FINSEQ_1, FINSEQ_2;
 expansions TARSKI, XBOOLE_0;
 theorems FINSEQ_1, FINSEQ_2, FUNCT_1, FUNCT_2, TARSKI, COMPLEX1, FUNCOP_1,
      ORDINAL1, NUMBERS, XBOOLE_1;

begin

reserve a,b,c,d for Real;
reserve x,y,x3,y3,X,z,Y,Z,V for set;

theorem :: MOMENTM1:1
  (REAL) c= COMPLEX;

theorem :: MOMENTM1:2 ::CCD20:
  <i> in COMPLEX;

theorem :: MOMENTM1:3 ::CCD21:
  <i>*<i> = -1r;

theorem :: MOMENTM1:4
  not <i> in REAL;

theorem :: MOMENTM1:5 ::CCC21:
  (REAL) c< COMPLEX;

theorem :: MOMENTM1:6
  REAL* is FinSequenceSet of REAL;

theorem :: MOMENTM1:7
  COMPLEX* is FinSequenceSet of COMPLEX;

theorem :: MOMENTM1:8
REAL* is FinSequenceSet of COMPLEX;

theorem :: MOMENTM1:9
  REAL* c= COMPLEX*;

definition
  let n be Nat;
  func COMPLEX n -> FinSequenceSet of COMPLEX equals
```

```
:: MOMENTM1:def 1
  n−tuples_on COMPLEX;
end;

registration
  let n be Nat;
  cluster COMPLEX n −> non empty;
end;

registration
  let n be Nat;
  cluster −> n−element for Element of COMPLEX n;
end;

theorem :: MOMENTM1:10
  for n being Nat holds
  REAL n = n−tuples_on REAL;

theorem :: MOMENTM1:11
  for n being Nat holds
  COMPLEX n = n−tuples_on COMPLEX;

registration
  let n be Nat;
  cluster −> n−element for Element of (COMPLEX n);
end;

theorem :: MOMENTM1:12
  for n being Nat holds
  REAL n = n−tuples_on REAL;

theorem :: MOMENTM1:13
  for n being Nat holds
  COMPLEX n = n−tuples_on COMPLEX;

theorem :: MOMENTM1:14
  for n being Nat holds
  REAL n is FinSequenceSet of REAL;

theorem :: MOMENTM1:15
  for n being Nat holds
  COMPLEX n is FinSequenceSet of COMPLEX;

registration
  let n be Nat;
  cluster COMPLEX n −> non empty;
end;

registration
  let n be Nat;
  cluster −> n−element for Element of COMPLEX n;
end;

theorem :: MOMENTM1:16 ::CCC22:
  for n being Nat holds
  COMPLEX n = n−tuples_on COMPLEX;

reserve z for Complex;

reserve x,x3,y,z,P,Q,X,Y,Z for set;

theorem :: MOMENTM1:17
  for n being Nat holds
  n−tuples_on REAL = Funcs(Seg n,REAL);

theorem :: MOMENTM1:18
  for n being Nat holds
  n−tuples_on COMPLEX = Funcs(Seg n,COMPLEX);

theorem :: MOMENTM1:19
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  for x being object,n being Nat holds
  x in Funcs(Seg n,REAL) iff ex f being Function st x = f &
  dom f = Seg n & rng f c= REAL;

theorem :: MOMENTM1:20
  for x being object,n being Nat holds
  x in Funcs(Seg n,COMPLEX) iff ex f being Function st x = f &
  dom f = Seg n & rng f c= COMPLEX;

theorem :: MOMENTM1:21
  for n being Nat holds
  n-tuples_on COMPLEX = Funcs(Seg n,COMPLEX);

theorem :: MOMENTM1:22
for n being Nat holds
  Funcs(Seg n,REAL)c= Funcs(Seg n,COMPLEX);

theorem :: MOMENTM1:23
  for n being Nat holds (n-tuples_on REAL)
  c= (n-tuples_on COMPLEX);

theorem :: MOMENTM1:24
  for n being Nat holds (REAL n) c= (COMPLEX n);

reserve f for real-valued FinSequence;

definition
  let n be Nat;
  func I*n -> FinSequence equals
:: MOMENTM1:def 2
  n |-> In(1,REAL);
end;

definition
  let n be Nat;
  func i*n -> complex-valued FinSequence equals
:: MOMENTM1:def 3
  n |-> In(<i>,COMPLEX);
end;

theorem :: MOMENTM1:25 ::CCC525:
  for n being Nat holds
  i*n is Element of COMPLEX n;

theorem :: MOMENTM1:26 ::CCC526:
  for n being Nat st 1 <= n
  holds i*n in COMPLEX n;

registration
  let n be Nat;
  cluster -> n-element for Element of COMPLEX n;
end;

theorem :: MOMENTM1:27
  for n being Nat st 1 <= n
  holds rng (i*n) = {<i>} &
  ex f2 being Function st
  dom f2 = Seg n & rng f2 c= COMPLEX;

theorem :: MOMENTM1:28
for n being Nat
  holds Funcs(Seg n,REAL)c= Funcs(Seg n,COMPLEX);

theorem :: MOMENTM1:29
for n being Nat st 1 <= n
  holds i* n in Funcs(Seg n,COMPLEX)
  & not i* n in Funcs(Seg n,REAL);

theorem :: MOMENTM1:30
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  for n being Nat st 1 <= n
  holds Funcs(Seg n,REAL)c< Funcs(Seg n,COMPLEX);

theorem :: MOMENTM1:31
  for n being Nat st n>=1
  holds (n-tuples_on REAL) c< (n-tuples_on COMPLEX);

theorem :: MOMENTM1:32
  for n being Nat st n>=1
  holds (REAL n) c< (COMPLEX n);
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Listing 2. MOMENTM1 - vocabulary (momentm1.voc)

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OI*
Oi*
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Listing 3. MOMENTM1 - article (momentm1.miz)

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:: On n-dimensional Real Spaces and n-dimensional Complex Spaces I
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environ

 vocabularies TARSKI, NUMBERS, NAT_1, FINSEQ_2, FINSEQ_1, SUBSET_1, SQUARE_1,
      VALUED_0, CARD_1, XXREAL_0, XCMPLX_0, FUNCT_1, FUNCT_2, FUNCT_7,
      XBOOLE_0, RVSUM_1, COMPLEX1, RELAT_1, REAL_1, ARYTM_1, MOMENTM1;
 notations TARSKI, XBOOLE_0, VALUED_0, FUNCT_1, FUNCT_2, ORDINAL1, CARD_1,
      NUMBERS, XCMPLX_0, XREAL_0, COMPLEX1, FINSEQ_1, FINSEQ_2, DOMAIN_1,
      RELAT_1, XXREAL_0, SUBSET_1, EUCLID;
 constructors COMPLEX1, FINSEQOP, TOPMETR, EUCLID;
 registrations RELSET_1, NUMBERS, XREAL_0, FINSEQ_2, RVSUM_1, ORDINAL1,
      COMPLEX1, XCMPLX_0;
 requirements NUMERALS, SUBSET, BOOLE;
 definitions TARSKI;
 equalities EUCLID, FINSEQ_1, FINSEQ_2;
 expansions TARSKI, XBOOLE_0;
 theorems FINSEQ_1, FINSEQ_2, FUNCT_1, FUNCT_2, TARSKI, COMPLEX1, FUNCOP_1,
      ORDINAL1, NUMBERS, XBOOLE_1;

begin

reserve a,b,c,d for Real;
reserve x,y,x3,y3,X,z,Y,Z,V for set;

theorem Th1:
  (REAL) c= COMPLEX by NUMBERS:11;

theorem ::CCD20:
  <i> in COMPLEX;

theorem ::CCD21:
  <i>*<i> = -1r by COMPLEX1:18;

theorem Th4:
  not <i> in REAL by COMPLEX1:7;

theorem ::CCC21:
  (REAL) c< COMPLEX by Th4,Th1;

theorem
  REAL* is FinSequenceSet of REAL;
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theorem
  COMPLEX* is FinSequenceSet of COMPLEX;

theorem Th8: REAL* is FinSequenceSet of COMPLEX
  by FINSEQ_2:91,Th1;

theorem
  REAL* c= COMPLEX* by FINSEQ_2:90,Th8;

definition
  let n be Nat;
  func COMPLEX n -> FinSequenceSet of COMPLEX equals
  n-tuples_on COMPLEX;
  coherence;
end;

registration
  let n be Nat;
  cluster COMPLEX n -> non empty;
  coherence;
end;

registration
  let n be Nat;
  cluster -> n-element for Element of COMPLEX n;
  coherence;
end;

theorem
  for n being Nat holds
  REAL n = n-tuples_on REAL;

theorem
  for n being Nat holds
  COMPLEX n = n-tuples_on COMPLEX;

registration
  let n be Nat;
  cluster -> n-element for Element of (COMPLEX n);
  coherence;
end;

theorem
  for n being Nat holds
  REAL n = n-tuples_on REAL;

theorem
  for n being Nat holds
  COMPLEX n = n-tuples_on COMPLEX;

theorem
  for n being Nat holds
  REAL n is FinSequenceSet of REAL;

theorem
  for n being Nat holds
  COMPLEX n is FinSequenceSet of COMPLEX;

registration
  let n be Nat;
  cluster COMPLEX n -> non empty;
  coherence;
end;

registration
  let n be Nat;
  cluster -> n-element for Element of COMPLEX n;
  coherence;
end;
```

```
theorem ::CCC22:
  for n being Nat holds
  COMPLEX n = n-tuples_on COMPLEX;

reserve z for Complex;

reserve x,x3,y,z,P,Q,X,Y,Z for set;

theorem Th17:
  for n being Nat holds
  n-tuples_on REAL = Funcs(Seg n,REAL) by FINSEQ_2:93;

theorem Th18:
  for n being Nat holds
  n-tuples_on COMPLEX = Funcs(Seg n,COMPLEX) by FINSEQ_2:93;

theorem Th19:
  for x being object,n being Nat holds
  x in Funcs(Seg n,REAL) iff ex f being Function st x = f &
  dom f = Seg n & rng f c= REAL by FUNCT_2:def 2;

theorem Th20:
  for x being object,n being Nat holds
  x in Funcs(Seg n,COMPLEX) iff ex f being Function st x = f &
  dom f = Seg n & rng f c= COMPLEX by FUNCT_2:def 2;

theorem Th21:
  for n being Nat holds
  n-tuples_on COMPLEX = Funcs(Seg n,COMPLEX) by FINSEQ_2:93;

theorem Th22:for n being Nat holds
  Funcs(Seg n,REAL)c= Funcs(Seg n,COMPLEX)
  proof
    let n be Nat;
    thus Funcs(Seg n,REAL)c= Funcs(Seg n,COMPLEX)
    proof
      let x3 be object;
      assume x3 in Funcs(Seg n,REAL);then
      consider f be Function such that
      A1: x3 = f &
      dom f = Seg n & rng f c= REAL by Th19;
      thus x3 in Funcs(Seg n,COMPLEX) by Th1,Th20,XBOOLE_1:1,A1; ::CCC20,
    end;
  end;

theorem Th23:
  for n being Nat holds (n-tuples_on REAL)
  c= (n-tuples_on COMPLEX)
  proof
    let n be Nat;
    now let x3 being object;
      assume A1: x3 in (n-tuples_on REAL);
      A2: Funcs(Seg n,REAL) c= Funcs(Seg n,COMPLEX) by Th22;
      x3 in Funcs(Seg n,REAL)by Th17,A1;then
      x3 in Funcs(Seg n,COMPLEX)by A2;
      hence x3 in (n-tuples_on COMPLEX)by Th21;
    end;
    hence thesis;
  end;

theorem
  for n being Nat holds (REAL n) c= (COMPLEX n) by Th23;

reserve f for real-valued FinSequence;

definition
  let n be Nat;
  func I*n -> FinSequence equals
  n |-> In(1,REAL);
```

```
  correctness;
end;

definition
  let n be Nat;
  func i*n -> complex-valued FinSequence equals
  n |-> In(<i>,COMPLEX);
  correctness;
end;

theorem ::CCC525:
  for n being Nat holds
  i*n is Element of COMPLEX n;

theorem ::CCC526:
  for n being Nat st 1 <= n
  holds i*n in COMPLEX n;

registration
  let n be Nat;
  cluster -> n-element for Element of COMPLEX n;
  correctness;
end;

theorem Th27:
  for n being Nat st 1 <= n
  holds rng (i*n) = {<i>} &
  ex f2 being Function st
  dom f2 = Seg n & rng f2 c= COMPLEX
  proof
    let n be Nat;
    assume A1: 1 <= n;
    A2: 1 in Seg n by A1;

    A3: n is Element of NAT by ORDINAL1:def 12;
    for A being set, p being FinSequence of
    A holds p in n -tuples_on A iff len p = n by FINSEQ_2:133,A3;then
    A4: for p being FinSequence of COMPLEX holds
    ( p in n -tuples_on COMPLEX iff len p = n );

    A5: i* n in n -tuples_on COMPLEX iff len (i* n) = n by A4;
    A6: len (i* n) = n by A5;
    A7: Seg len (i* n) = dom (i* n) by FINSEQ_1:def 3;
    A8: len (i* n) =n by A6;then
    A9: dom (i*n) = Seg n by A7;
    1 in { k where k is Nat : ( 1 <= k & k <= n ) } by A2;then
    A10: 1 in Seg n;then
    A11: 1 in dom (i*n) by A9;
    A12: (n |-> In(<i>,COMPLEX)).1 = In(<i>,COMPLEX)by A10,FUNCOP_1:7;

    reconsider p = (n |-> In(<i>,COMPLEX)) as FinSequence;
    A13: Seg len p = dom p by FINSEQ_1:def 3;
    p.1 = In(<i>,COMPLEX) by A12; then
    A14: (n |-> <i>).1
    = In(<i>,COMPLEX)
    .= <i> ;
    A15: (n |-> In(<i>,COMPLEX)).1= <i> by A14;
    rng (n |-> In(<i>,COMPLEX))= {<i>}
    proof
      A16: rng(n |-> In(<i>,COMPLEX)) c= {<i>}
      proof let y3 be object;
        assume y3 in rng(n |-> In(<i>,COMPLEX));then
        consider x3 being object such that
        A17: x3 in dom (n |-> In(<i>,COMPLEX))
        & (n |-> In(<i>,COMPLEX)).x3 = y3 by FUNCT_1:def 3;
        x3 in dom (n |-> In(<i>,COMPLEX)) by A17;then
        x3 in Seg n by A13,A8;then
        x3 in Seg n ;then
        A18: (n |-> In(<i>,COMPLEX)).x3 = In(<i>,COMPLEX)by FUNCOP_1:7;
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        (n |-> <i>).x3
        = In(<i>,COMPLEX) by A18
        .= <i> ;
        then
        A19: (n |-> In(<i>,COMPLEX)).x3= <i>;
        A20: x3 in dom (n |-> In(<i>,COMPLEX))
        & (n |-> In(<i>,COMPLEX)).x3= y3 by A17;
        y3= <i> by A20,A19;
        hence y3 in {<i>} by TARSKI:def 1;
      end;

      {<i>} c= rng(n |-> In(<i>,COMPLEX))
      proof let y3 be object;
        assume y3 in {<i>};then
        A21: y3= <i> by TARSKI:def 1;
        A22: 1 in dom (n |-> In(<i>,COMPLEX)) by A11;
        1 in dom (n |-> In(<i>,COMPLEX))
        & (n |-> In(<i>,COMPLEX)).1= y3 by A21,A22,A15;
        hence y3 in rng(n |-> In(<i>,COMPLEX)) by FUNCT_1:def 3;
      end;
      hence thesis by A16;
    end;
    hence rng (i*n) = {<i>} &
    ex f2 being Function st
    dom f2 = Seg n & rng f2 c= COMPLEX by A9;
  end;

theorem Th28:
  for n being Nat
  holds Funcs(Seg n,REAL)c= Funcs(Seg n,COMPLEX) by Th22;

theorem Th29:
  for n being Nat st 1 <= n
  holds i* n in Funcs(Seg n,COMPLEX)
  & not i* n in Funcs(Seg n,REAL)
  proof let n be Nat;
    assume A1: 1 <= n;
    n-tuples_on COMPLEX = Funcs(Seg n,COMPLEX) by FINSEQ_2:93;then
    A2: i* n in Funcs(Seg n,COMPLEX);
    1 in { k where k is Nat: 1 <= k & k <= n } by A1;then
    A3: 1 in Seg n;
    A4: not <i> in REAL by Th4;
    now assume A5: i* n in Funcs(Seg n,REAL);
      consider f be Function such that
      A6: (i* n) = f & dom f = Seg n & rng f c= REAL by A5,FUNCT_2:def 2;
      1 in Seg n by A3;then
      A7: (i* n).1 in rng (i* n) by A6,FUNCT_1:def 3;
      (i*n).1 in {<i>} by A7,A1,Th27;then
      A8: (i*n).1 = <i> by TARSKI:def 1;
      <i> in REAL by A8,A7,A6;
      hence contradiction by A4;
    end;then
    not i* n in Funcs(Seg n,REAL);
    hence thesis by A2;
  end;

theorem Th30:
  for n being Nat st 1 <= n
  holds Funcs(Seg n,REAL)c< Funcs(Seg n,COMPLEX)
  proof let n be Nat;
    assume A1: 1 <= n;
    A2: Funcs(Seg n,REAL)c= Funcs(Seg n,COMPLEX)by Th28;
    now assume A3: Funcs(Seg n,REAL)= Funcs(Seg n,COMPLEX);
      i* n in Funcs(Seg n,COMPLEX) by Th29,A1;
      hence contradiction by A3,Th29,A1;
    end;
    hence thesis by A2;
  end;
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theorem Th31:
  for n being Nat st n>=1
  holds (n-tuples_on REAL) c< (n-tuples_on COMPLEX)
  proof let n be Nat;
    assume A1: n>=1;
    A2: Funcs(Seg n,REAL)=(n-tuples_on REAL) by Th17;
    Funcs(Seg n,COMPLEX)=(n-tuples_on COMPLEX) by Th18;
    hence thesis by A1,A2,Th30;
  end;

theorem
  for n being Nat st n>=1
  holds (REAL n) c< (COMPLEX n) by Th31;
```