Regular Paper

二等辺三角形型および等脚台形型メンバシップ関数の重心法を用いた非ファジィ化値

Centroids of Isosceles Triangular and Isosceles Trapezoidal Membership Functions

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Abstract

The centroids of isosceles triangular and trapezoidal membership functions are mentioned in this paper. We proved the agreement of the same function expressed with different parameters and formalized those centroid with those parameters. In addition, various properties of membership functions on intervals where the endpoints of the domain are fixed and on general intervals are formalized.

1 はじめに

ファジィ集合を表すメンバシップ関数の形状は，大別すると z 型，π 型，s 型の 3 種類がある．そのうち π 型には，直線のみで構成される三角型，台形型などがあり，シグモイド関数やガウス関数などの曲線で構成される釣り鐘型と呼ばれる形状も提案されている．

設計者の主観で決定する場合が多い前件部変数値に対するメンバシップ関数値の妥当性の判断は困難であり，そのため IF-THEN ルールをいかなる形状の関数で構成するかは，ファジィ最適化問題としてさまざまな研究がなされてきた [1] [2]．そのような背景により，従来実用においては，推論計算の容易さから三角型，台形型のメンバシップ関数が適用されてきた．そして左右非対称な形状に必要性がない場合には，左右対称な二等辺三

角形，等脚台形が多く用いられている．そこで本研究では，これらのメンバシップ関数の形式化とともに重心法による非ファジィ化値の形式化を行った．

非ファジィ化の手続きは通常推論結果に対して施すものであるが，和算やＭＡＸ演算で複数のメンバシップ関数を計算したファジィ推論結果の関数が二等辺三角形，等脚台形になることは稀である．しかしながら Center of Sums 法，面積法，高さ法，密度モーメント法では，推論計算（非ファジィ化）において IF-THEN ルールを構成するファジィ集合のメンバシップ関数の重心（非ファジィ化値）を用いている [3–5]．

2　非ファジィ化値の存在区間

後述のメンバシップ関数の重心法による非ファジィ化値は，一般的な表現を用いると，座標平面の横軸上にある図形の底辺に該当する部分の区間に存在する．

メンバシップ関数値が 0 となる，いわゆる関数のグラフによる図形の外側にあたる定義域には非ファジィ化値は存在しない．そこで次の 2 つの定理はそれらの区間を関数の定義域から除外することが可能であることを証明している．後述する定理で示す三角形型や台形型メンバシップ関数の centroid（重心法による非ファジィ化値）の計算 [6] において，非ファジィ化の区間を三角形，台形の底にあたる区間としている．これらの証明に用いるため以下の定理は，非ファジィ化されるメンバシップ関数の定義域を非ファジィ化値が存在する区間に限定できることを表している．

Listing 1. FUZZY_7:17, FUZZY_7:21

```
theorem :: FUZZY_7:17
for a,b,c,d be Real, f be Function of REAL,REAL st
a < b & b < c & c < d & f is_integrable_on ['a,d'] & f | ['a,d'] is bounded & for x be Real st x in ['a,b'] \/ ['c,d']
holds f.x = 0 holds centroid(f,['a,d']) = centroid(f,['b,c']);

theorem :: FUZZY_7:21
for A,B be non empty closed_interval Subset of REAL, f be Function of REAL,REAL st
lower_bound B <> upper_bound B & B c= A & f is_integrable_on A & f | A is bounded &
(for x be Real st x in (A \ B) holds f.x = 0) & f.(lower_bound B) = 0 & f.(upper_bound B) = 0 holds
centroid (f,A) = centroid (f,B);
```

つまり上記 Listing 1 は，$b \in [a, c]$ とし，閉区間 $[a, c]$ の部分集合 $[b, c]([a, b])$ においてメンバシップ関数 f が $f(x) = 0$ であるとき，この区間には重心値が存在せず，その区間を省いて非ファジィ化値を計算する区間を $[a, b]([b, c])$ とできることを表している．

3　二等辺三角形型および等脚台形型メンバシップ関数

article [7] において，図 1 のような三角型，台形型メンバシップ関数は関数値が 1 または 0 となる定義域の値をパラメータとして以下のように TriangularFS，TrapezoidalFS の形式で定義されている．

Listing 2. FUZNUM_1:def 7, 8

```
definition
let a, b, c be Real;
assume that
Z1: a < b and
Z2: b < c ;
```

```
func TriangularFS (a,b,c) −> FuzzySet of REAL equals :TrDef: :: FUZNUM_1:def 7
(((AffineMap (0,0)) | (REAL \ ].a,c.[)) +* ((AffineMap ((1 / (b − a)),(− (a / (b − a))))) | [.a,b.]))
+* ((AffineMap ((− (1 / (c − b))),(c / (c − b)))) | [.b,c.]);
coherence
proof end;
end;

definition
 let a, b, c, d be Real;
 assume that
 Z1: a < b and
 Z2: b < c and
 Z3: c < d ;
 func TrapezoidalFS (a,b,c,d) −> FuzzySet of REAL equals :TPDef: :: FUZNUM_1:def 8
 ((((AffineMap (0,0)) | (REAL \ ].a,d.[)) +* ((AffineMap ((1 / (b − a)),(− (a / (b − a))))) | [.a,b.]))
 +* ((AffineMap (0,1)) | [.b,c.])) +* ((AffineMap ((− (1 / (d − c))),(d / (d − c)))) | [.c,d.]);
 coherence
 proof end;
end;
```

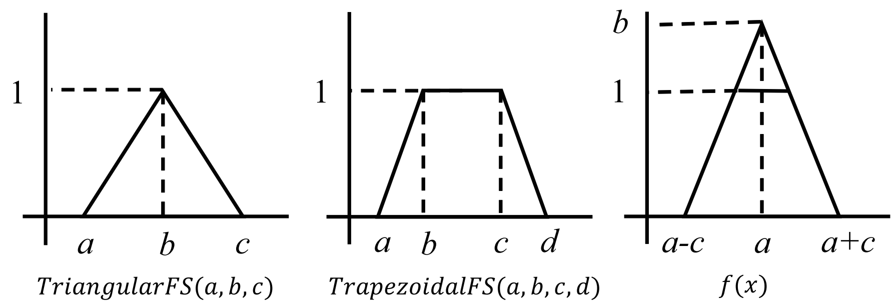

図 1. Triangular Membership Functions and Trapezoidal Membership Function

Listing 3. FUZNUM_1:def 9

```
definition
 let F be FuzzySet of REAL;
 attr F is triangular means :: FUZNUM_1:def 9
 ex a, b, c being Real st F = TriangularFS (a,b,c);
 attr F is trapezoidal means :: FUZNUM_1:def 10
 ex a, b, c, d being Real st F = TrapezoidalFS (a,b,c,d);
end;
```

本研究では MAX 演算と絶対値記号を用いて，図 1 のような座標平面上において底辺の長さが $2c(c > 0)$ で，頂点の座標が (a, b) である二等辺三角形型の関数 $f(x)$ を形式化した．

$$f(x) = \max\left\{0, b - \left|\frac{b(x-a)}{c}\right|\right\} \tag{1}$$

式 (1) の絶対値の内側の関数は以下のように区分線形関数 [8] として表した．

Listing 4. FUZZY_7:4

```
theorem :: FUZZY_7:4
for a,b,c be Real st b > 0 & c > 0 holds
for x be Real holds
( AffineMap ( b/c,b−a*b/c) | ]. −infty,a .]+* AffineMap (−b/c,b+a*b/c) | [. a,+infty .[ ).x = b − |. b*(x−a)/c .|;
```

さらに区分的線形関数は Mizar で積分を形式化するうえで比較的使いやすい AffineMap を用いる [9].

以下の定理は，上述の二等辺三角形型の関数の頂点の座標が $(a, 1)$ であるとき，三角型メンバシップ関数であることを示している．つまり上述の Listing 3 を満足している．

Listing 5. FUZZY_7:3

```
theorem :: FUZZY_7:3
for a,c be Real, f be Function of REAL,REAL st c > 0 & ( for x be Real holds f.x = max(0, 1 − |. (x−a)/c .|) )
holds f is FuzzySet of REAL & f is triangular FuzzySet of REAL;
```

一方，台形型メンバシップ関数は三角形型メンバシップ関数に MIN 関数による頭切り (clipping method) を適用して表現が可能である．式 (1) で示される二等辺三角形メンバシップ関数に MIN 関数を使って関数値 1 で頭切りをすると台形型メンバシップ関数の条件を満足する．

Listing 6. FUZZY_7:6

```
theorem :: FUZZY_7:6
for a,b,c be Real, f be Function of REAL,REAL st
b > 1 & c > 0 & ( for x be Real holds f.x = min(1, max(0, b − |. b∗(x−a)/c .|)) ) holds
f is FuzzySet of REAL & f is trapezoidal FuzzySet of REAL & f is normalized FuzzySet of REAL;
```

4　Lipschitz 連続性

筆者らはファジィ制御の最適制御存在性の証明において，ファジィ推論結果の前件部変数に関する Lipschitzian 連続性の考察を行った [10,11]．以下の 2 定理は，今後それらの証明の Mizar による検証に適用される見込みである．1 つめは本稿で形式化したメンバシップ関数に対するもので，他方は区分的線形関数に対するものである．

Listing 7. FUZZY_7:24, FUZZY_7:38

```
theorem :: FUZZY_7:24
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 & (for x be Real holds f.x = max(0,b−|. b∗(x−a)/c .|)) holds
f is Lipschitzian;

theorem :: FUZZY_7:38
for a,b,p,q be Real, f be Function of REAL,REAL st a <> p &
f = (AffineMap (a,b)|].−infty,(q−b)/(a−p).]) +∗ (AffineMap (p,q)|[.(q−b)/(a−p),+infty.[) holds
f is Lipschitzian;
```

5　二等辺三角形型および等脚台形型関数の重心

明らかに二等辺三角形型関数の重心法による非ファジィ化値はその頂点から底辺に下した垂線の足の座標である．以下の定理でその事実を形式化している．

Listing 8. FUZZY_7:25, FUZZY_7:26, FUZZY_7:27

```
theorem :: FUZZY_7:25
```

```
for a,b,c be Real, f be Function of REAL,REAL st b > 0 & c > 0 &
f | ['a−c,a+c']
= ( AffineMap ( b/c,b−a*b/c) |[. lower_bound ['a−c,a+c'], ((b+a*b/c) − (b−a*b/c))/((b/c)−(−b/c)) .] )
+* ( AffineMap (−b/c,b+a*b/c) |[. (b+a*b/c − (b−a*b/c))/((b/c)−(−b/c)), upper_bound ['a−c,a+c'] .] )
holds centroid (f,['a−c,a+c']) = a;

theorem :: FUZZY_7:26
for a,b,c be Real, f be Function of REAL,REAL st b > 0 & c > 0 &
( for x be Real holds f.x = max(0, b − |. b*(x−a)/c .|) ) holds
f | ['a−c,a+c'] =
( AffineMap ( b/c,b−a*b/c) | [. lower_bound ['a−c,a+c'], (b+a*b/c − (b−a*b/c))/((b/c)−(−b/c)) .] )
+* ( AffineMap (−b/c,b+a*b/c) | [. (b+a*b/c − (b−a*b/c))/((b/c)−(−b/c)), upper_bound ['a−c,a+c'] .] );

theorem :: FUZZY_7:27
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 & ( for x be Real holds f.x = max(0,b−|. b*(x−a)/c .|) ) holds centroid (f,['a−c,a+c']) = a;
```

上の定理は頂点の座標が (a, b) の二等辺三角形型の関数の重心を求めている．これについて，$b = 1$ とすることによってメンバシップ関数として扱うことができる．最初の定理で区分的線形関数で表された二等辺三角形型メンバシップ関数の重心が a であることを証明し，次の定理でその関数が（1）の形に変換されること用いて証明された．

以下の定理は Listing 2 の形式で表される二等辺三角形型メンバシップ関数の重心が b であることを示している．

Listing 9. FUZZY_7:31, FUZZY_7:33

```
theorem :: FUZZY_7:31
for a,b,c be Real st a < b & b < c & b−a = c−b holds centroid (TriangularFS (a,b,c),['a,c']) = b;

theorem :: FUZZY_7:33
for a,b,c,d be Real st a < b & b < c & b−a = c−b & d <> 0 holds centroid (d (#) TriangularFS (a,b,c),['a,c']) = b;
```

等脚台形型メンバシップ関数の重心が上底（下底）の中点であることは自明である．以下の定理はその事実を形式化した．

Listing 10. FUZZY_7:36, FUZZY_7:48, FUZZY_7:49

```
theorem :: FUZZY_7:36
for a1,c,a2,d be Real, f be Function of REAL,REAL st
c > 0 & d > 0 & a1 < a2 & f = ( d (#) TrapezoidalFS (a1−c,a1,a2,a2+c) ) | ['a1−c,a2+c']
holds f is_integrable_on ['a1−c,a2+c'];

theorem :: FUZZY_7:48
for a1,c,a2,d be Real, f be Function of REAL,REAL st c > 0 & d > 0 & a1 < a2 &
f | [.a1−c,a2+c.] = AffineMap(d/c,−(d/c)*(a1 − c)) | [.a1−c,a1.] +*
                    AffineMap(0,d) | [.a1 ,a2.] +*
                    AffineMap(−d/c, (d/c)*(a2 + c)) | [.a2,a2+c.]
holds
integral( f,['a1−c,a2+c']) =
integral( AffineMap(d/c,−(d/c)*(a1 − c)), ['a1− c,a1']) +
integral( AffineMap(0,d) , ['a1 ,a2']) +
integral( AffineMap(−d/c, (d/c)*(a2 + c)) , ['a2,a2+c']) ;

theorem :: FUZZY_7:49
for a1,c,a2,d be Real st c > 0 & d > 0 & a1 < a2 holds
centroid ( d (#) TrapezoidalFS (a1−c,a1,a2,a2+c), ['a1−c,a2+c']) = (a1+a2)/2;
```

ここで用いられている等脚台形型関数は上底の長さが $a2-a1$，下底の長さが $a2-a1+2c$，

高さが d である．その重心は $\dfrac{a1+a2}{2}$ である．二等辺三角形型と同様で，特に $d=1$ とすればメンバシップ関数となる．

6 まとめ

本研究では，二等辺三角形型および等脚台形型メンバシップ関数の重心法による非ファジィ化値の形式化を行った．いずれの重心も対象の軸と定義域との交点の座標であることは自明であるが，関数の積分やメンバシップ関数の定義域などの関連定理は今後の非ファジィ化計算の形式化に役に立つことが期待できる．メンバシップ関数に限らず，π 型の関数の形状が対象であるという概念の一般化や形式化は今後の課題としたい．さらに，本稿により後件部のメンバシップ関数の和算・MAX 演算を伴わない Center of Sums 法などのファジィ推論の入力値に対する連続性の検証が可能になった．こちらの Mizar による検証も今後の課題としたい．最後に，対称（線対象，点対称など）の概念の形式化が今後必要かの議論を課題としたい．

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Mizar article information

Works in Progress

FUZZY_7 Isosceles Triangular and Isosceles Trapezoidal Membership Functions
by Takashi Mitsuishi

Summary: The centroids of isosceles triangular and isosceles trapezoidal membership functions are mentioned in this paper. We proved the agreement of the same function expressed with different parameters and formalized those centroid with those parameters. In addition, various properties of membership functions on intervals where the endpoints of the domain are fixed and on general intervals are formalized.

Listing 11. FUZZY_7 - abstract

```
environ

 vocabularies NUMBERS, XBOOLE_0, SUBSET_1, XXREAL_1, CARD_1, RELAT_1, TARSKI,
      FUNCT_1, XXREAL_0, PARTFUN1, ARYTM_1, ARYTM_3, COMPLEX1,
       MEASURE5, FUZNUM_1, REAL_1, ORDINAL2, XXREAL_2,FUNCT_7,
      JGRAPH_2, FUNCT_4, FCONT_1,SQUARE_1,SIN_COS,
      VALUED_1,FDIFF_1, INTEGRA1,INTEGRA5,SEQ_4,
POLYEQ_3,NEWTON, MSALIMIT,NUMPOLY1, FUZZY_6;

 notations
TARSKI, XBOOLE_0, SUBSET_1, NEWTON, RELAT_1,
      ORDINAL1,NUMBERS, SQUARE_1, BINOP_1, NORMSP_1,METRIC_1,
       INT_1, XCMPLX_0,XXREAL_0, COMPLEX1,XXREAL_2,XREAL_0,
REAL_1, RELSET_1,
PBOOLE, MEASURE5, PARTFUN1, FUNCT_2,
PARTFUN2, FUNCT_1, FCONT_1,
FUNCT_3,INTEGRA1,INTEGRA2, RFUNCT_3,
FUNCOP_1, NAT_1, VALUED_0, VALUED_1, FUNCT_4,
ARYTM_2, ARYTM_1, ARYTM_0, COMSEQ_2, FDIFF_1,
 MEMBERED, FINSEQ_1, FINSEQ_2, SEQ_2, SEQ_4,
```

SIN_COS, RFUNCT_1, POLYEQ_3,
RVSUM_1,RCOMP_1, TOPMETR, TAYLOR_1, PRE_TOPC, TOPS_2, POLYDIFF,
INTEGRA5,COUSIN2, DOMAIN_1, TSEP_1, TMAP_1,
FUZZY_1,FUZNORM1,FUZNUM_1, FUZZY_6;

theorems
TARSKI, XBOOLE_0, FUNCT_1, ABSVALUE,
COMPLEX1, XREAL_1, XXREAL_0, XXREAL_1, FUNCT_2, RELAT_1,
XREAL_0, FCONT_1, XCMPLX_1, FUNCT_4,
FDIFF_1,VALUED_1,INTEGRA1,INTEGRA5,
NEWTON,INTEGRA4, INTEGRA6,SEQ_4, COUSIN2,
POLYEQ_3, FUZZY_5, FUZZY_6,FUZNUM_1;

constructors
XBOOLE_0, ZFMISC_1, SUBSET_1, FUNCT_1, FUNCT_2, XCMPLX_0,
XXREAL_0, XREAL_0,
COMPLEX1, RFUNCT_1, SEQ_4, RELSET_1, FUZZY_1, FCONT_1,
FUNCT_4,FUZNUM_1,
LFUZZY_1,RELAT_1,SIN_COS,XXREAL_3,NUMBERS,
SIN_COS6,TOPMETR,SQUARE_1,TAYLOR_1,POWER, COMPTS_1, RCOMP_1,
FINSEQ_1, FINSEQ_2, INT_1,ORDINAL1,
FDIFF_1, NORMSP_1,METRIC_1,VALUED_1,PARTFUN1,MEASURE5,
COMSEQ_2, PARTFUN2,POLYDIFF,PREPOWER,NEWTON,
RFUNCT_3,FUZNORM1, BINOP_1, BINOP_2,INTEGRA1,INTEGRA2,INTEGRA3,INTEGRA5,
COUSIN2,SEQ_1,SEQ_2,RVSUM_1, NAT_1,
XXREAL_2, EXTREAL1, MESFUNC5,TOPS_2, CONNSP_1,
TMAP_1, PCOMPS_1,REAL_1,ARYTM_1,ARYTM_0,PBOOLE,FUNCOP_1,FUNCT_3,
POLYEQ_3,FUZZY_5,FUZZY_6,MEMBERED;

registrations RELSET_1, NUMBERS, XXREAL_0, MEMBERED, XBOOLE_0, VALUED_0,
VALUED_1,COMSEQ_2,PDIFFEQ1,FDIFF_1,FUZZY_1,
FUNCT_2,XREAL_0, ORDINAL1, FCONT_1, RELAT_1, TOPREALB, FUNCT_4, FUNCT_1,
XCMPLX_0, NAT_1, RCOMP_1,FUZNUM_1,NUMPOLY1,
LFUZZY_1,SIN_COS,XXREAL_3,
SIN_COS6,TOPMETR,SQUARE_1,SIN_COS3, CARD_3, INT_1,SUBSET_1,SIN_COS9,
NORMSP_1,PARTFUN1,INTEGRA1,MEASURE5,COMSEQ_1,
FDIFF_2,POLYDIFF,PREPOWER,NEWTON,RFUNCT_3,FUZNORM1,INTEGRA2,RFUNCT_1,
SEQ_2,SEQ_1,RVSUM_1,NAT_3, PRE_TOPC, METRIC_1, BORSUK_1, CONNSP_1,
EXTREAL1, SUPINF_1, SUPINF_2 ,XXREAL_1;

requirements NUMERALS, REAL, SUBSET, BOOLE, ARITHM ;

definitions XBOOLE_0, TARSKI, XXREAL_2,FUZNUM_1,
FUZZY_1,RELAT_1,
XXREAL_3,NUMBERS,SIN_COS6,TOPMETR,SQUARE_1, COMPTS_1, RCOMP_1,
FCONT_1,INT_1,SUBSET_1,VALUED_1,XXREAL_0,SIN_COS,
PARTFUN1,INTEGRA1,INTEGRA2,INTEGRA3,INTEGRA5,MEASURE5,FUNCT_2,RFUNCT_1,
BINOP_1,FUZNORM1,SEQ_1,SEQ_2,COMSEQ_2,FINSEQ_1,NAT_1,TAYLOR_1,FUZZY_6,
RVSUM_1;

equalities FUZZY_1, RCOMP_1,FUZNUM_1,
RELAT_1,XXREAL_3,
NUMBERS,SIN_COS6,SQUARE_1,FUNCT_4,RFUNCT_1,INTEGRA1,INTEGRA2;

expansions TARSKI, FCONT_1,FUZNUM_1,
FUZZY_1,RELAT_1,XXREAL_3,
NUMBERS,SQUARE_1,FUNCT_4,RFUNCT_1,INTEGRA1,INTEGRA2;

schemes FUNCT_2;

begin

reserve A **for non** empty closed_interval Subset **of** REAL;

theorem *:: FUZZY_7:1*
for f,g **be** Function **of** REAL,REAL **st**
f **is** continuous **&** g **is** continuous

```
holds
max(f,g) is continuous;

theorem :: FUZZY_7:2
for f,g be Function of REAL,REAL st
f is continuous & g is continuous
holds
min(f,g) is continuous;

theorem :: FUZZY_7:3
for a,c be Real, f be Function of REAL,REAL st
c > 0 & ( for x be Real holds f.x = max(0, 1 - |. (x-a)/c .|) )
holds
f is FuzzySet of REAL & f is triangular FuzzySet of REAL;

theorem :: FUZZY_7:4
for a,b,c be Real st b > 0 & c > 0
holds
for x be Real holds
( AffineMap ( b/c,b-a*b/c) | ]. -infty,a .]
+* AffineMap (-b/c,b+a*b/c) | [. a,+infty .[ ).x = b - |. b*(x-a)/c .|;

theorem :: FUZZY_7:5
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 &
(for x be Real holds f.x = b-|. b*(x-a)/c .|) holds
f = AffineMap (b/c,b-a*b/c) | ]. -infty,a .]
 +* AffineMap (-b/c,b+a*b/c) | [. a,+infty .[;

theorem :: FUZZY_7:6
for a,b,c be Real, f be Function of REAL,REAL st
b > 1 & c > 0 &
( for x be Real holds f.x = min(1, max(0, b - |. b*(x-a)/c .|)) )
holds
f is FuzzySet of REAL &
f is trapezoidal FuzzySet of REAL &
f is normalized FuzzySet of REAL;

theorem :: FUZZY_7:7
for s be Real, f,g being Function of REAL,REAL holds
dom ( ( f | ].-infty,s.] ) +* ( g | [.s,+infty.[ ) ) = REAL &
dom ( ( f | ].-infty,s.[ ) +* ( g | [.s,+infty.[ ) ) = REAL;

theorem :: FUZZY_7:8
for a,b being Real holds
( a > 0 implies
|. AffineMap (a,b) .| = (- (AffineMap(a,b) | ].-infty,(-b)/a.[))
 +* (AffineMap(a,b) | [.(-b)/a,+infty.[) );

theorem :: FUZZY_7:9
for a,b being Real holds
( a < 0 implies
|. AffineMap (a,b) .| = ( (AffineMap(a,b) | ].-infty,(-b)/a.[ ))
+* - (AffineMap(a,b) | [.(-b)/a,+infty.[ ) );

theorem :: FUZZY_7:10
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 &
(for x be Real holds f.x = max(0, b - |. b*(x-a)/c .|)) holds
for x be Real st not x in ['a-c,a+c'] holds f.x = 0;

theorem :: FUZZY_7:11
for a,b,c be Real, f,g be Function of REAL,REAL st
a < b & b < c holds
(f | ]. -infty,b .] +* g | [. b,+infty .[ ) | [. a,c .]
= (f | [. a,b .]) +* (g | [. b,c .]);

theorem :: FUZZY_7:12
for a,b,c be Real, f be Function of REAL,REAL st
```

b > 0 **&** c > 0
holds
(AffineMap (b/c,b−a∗b/c) |]. −infty,a .]
+∗ AffineMap (−b/c,b+a∗b/c) | [. a,+infty .[ ) | [. a−c,a+c .]
= AffineMap (b/c,b−a∗b/c) | [. a−c,a .]
+∗ AffineMap (−b/c,b+a∗b/c) | [. a,a+c .];

theorem *:: FUZZY_7:13*
for a,b,c,d **be** Real **st** a < b **&** b < c **&** c < d **holds**
['a,d'] \ ['b,c'] c= ['a,b'] \/ ['c,d'];

theorem *:: FUZZY_7:14*
for a,b,c,d **be** Real **st** a < b **&** b < c **&** c < d **holds**
[. a,d .] \ [. b,c .] c= [. a,b .] \/ [. c,d .];

theorem *:: FUZZY_7:15*
for a,b,c,d **be** Real **st** a < b **&** b < c **&** c < d **holds**
[. a,d .] \ [. b,c .] = [. a,b .[\/]. c,d .];

theorem *:: FUZZY_7:16*
for a,b,c **be** Real, f **be** Function **of** REAL,REAL **st**
a < b **&** b < c **&** f is_integrable_on ['a,c'] **&** f | ['a,c'] **is** bounded
holds
f is_integrable_on ['a,b'] **&** f is_integrable_on ['b,c'] **&**
f | ['a,b'] **is** bounded **&** ['a,b'] c= dom f **&**
integral (f,a,c) = (integral (f,a,b)) + (integral (f,b,c));

theorem *:: FUZZY_7:17*
for a,b,c,d **be** Real, f **be** Function **of** REAL,REAL **st**
a < b **&** b < c **&** c < d **&**
f is_integrable_on ['a,d'] **&** f | ['a,d'] **is** bounded **&**
for x **be** Real **st** x in ['a,b'] \/ ['c,d'] **holds** f.x = 0
holds
centroid(f,['a,d']) = centroid(f,['b,c']);

theorem *:: FUZZY_7:18*
for p, q, r, s **being** Real **st** p < r **&** r <= s **&** s < q
holds
[.r,s.] c< [.p,q.];

theorem *:: FUZZY_7:19*
for A,B **be** **non** empty closed_interval Subset **of** REAL
st B c< A **holds**
lower_bound A < lower_bound B **or**
upper_bound B < upper_bound A;

theorem *:: FUZZY_7:20*
for A,B **be** **non** empty closed_interval Subset **of** REAL **st**
B c= A **holds**
lower_bound A <= lower_bound B **&** upper_bound B <= upper_bound A;

theorem *:: FUZZY_7:21*
for A,B **be** **non** empty closed_interval Subset **of** REAL,
f **be** Function **of** REAL,REAL **st** lower_bound B <> upper_bound B **&** B c= A **&**
f is_integrable_on A **&** f | A **is** bounded **&**
(**for** x **be** Real **st** x in (A \ B) **holds** f.x = 0) **&**
f.(lower_bound B) = 0 **&** f.(upper_bound B) = 0
holds
centroid (f,A) = centroid (f,B);

theorem *:: FUZZY_7:22*
for a,b,c **be** Real **st** c >= 0 **holds**
c ∗ max (a, b) = max (c∗a, c∗b) **&** c ∗ min (a, b) = min (c∗a, c∗b);

theorem *:: FUZZY_7:23 ::Th13:*
for a,b,c **be** Real, f **be** Function **of** REAL,REAL **st**
b > 0 **&** c > 0 **&**
(**for** x **be** Real **holds** f.x = max(0, b − |. b∗(x−a)/c .|)) **holds**
f = b **(#)** (TriangularFS (a−c,a,a+c));

```
theorem :: FUZZY_7:24
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 & (for x be Real holds f.x = max(0,b-|. b*(x-a)/c .|)) holds
f is Lipschitzian;

theorem :: FUZZY_7:25
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 &
f | ['a-c,a+c'] =
( AffineMap ( b/c,b-a*b/c) |
[. lower_bound ['a-c,a+c'], ((b+a*b/c) - (b-a*b/c))/((b/c)-(-b/c)) .] )
+* ( AffineMap (-b/c,b+a*b/c) |
[. (b+a*b/c - (b-a*b/c))/((b/c)-(-b/c)), upper_bound ['a-c,a+c'] .] )
holds
centroid (f,['a-c,a+c']) = a;

theorem :: FUZZY_7:26
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 &
( for x be Real holds f.x = max(0, b - |. b*(x-a)/c .|) ) holds
f | ['a-c,a+c'] =
( AffineMap ( b/c,b-a*b/c) |
[. lower_bound ['a-c,a+c'], (b+a*b/c - (b-a*b/c))/((b/c)-(-b/c)) .] )
+* ( AffineMap (-b/c,b+a*b/c) |
[. (b+a*b/c - (b-a*b/c))/((b/c)-(-b/c)), upper_bound ['a-c,a+c'] .] );

theorem :: FUZZY_7:27
for a,b,c be Real, f be Function of REAL,REAL st b > 0 & c > 0 &
( for x be Real holds f.x = max(0,b-|. b*(x-a)/c .|) ) holds
centroid (f,['a-c,a+c']) = a;

theorem :: FUZZY_7:28
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 &
(for x be Real holds f.x = max(0,b-|. b*(x-a)/c .|))
holds
f is_integrable_on A & f | A is bounded;

theorem :: FUZZY_7:29
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 &
(for x be Real holds f.x = max(0,b-|. b*(x-a)/c .|))
holds
f.(lower_bound ['a-c,a+c']) = 0 & f.(a-c) = 0 &
f.(upper_bound ['a-c,a+c']) = 0 & f.(a+c) = 0;

theorem :: FUZZY_7:30 ::Th20:
for a,b,c be Real, f be Function of REAL,REAL st
b > 0 & c > 0 & ['a-c,a+c'] c= A &
(for x be Real holds f.x = max(0,b-|. b*(x-a)/c .|))
holds
centroid (f,A) = a;

theorem :: FUZZY_7:31
for a,b,c be Real st a < b & b < c & b-a = c-b holds
centroid (TriangularFS (a,b,c),['a,c']) = b;

theorem :: FUZZY_7:32
for a,b,c be Real st a < b & b < c
holds
TriangularFS (a,b,c) is_integrable_on A &
TriangularFS (a,b,c) | A is bounded;

theorem :: FUZZY_7:33
for a,b,c,d be Real st a < b & b < c & b-a = c-b & d <> 0 holds
centroid (d (#) TriangularFS (a,b,c),['a,c']) = b;
```

```
theorem :: FUZZY_7:34
for a,b,c,d be Real st a < b & b < c & c < d
holds
TrapezoidalFS (a,b,c,d) is_integrable_on A &
TrapezoidalFS (a,b,c,d) | A is bounded;

theorem :: FUZZY_7:35
for a,b,c,d,r be Real st a < b & b < c & c < d
holds
r (#) TrapezoidalFS (a,b,c,d) is_integrable_on A;

theorem :: FUZZY_7:36
for a1,c,a2,d be Real, f be Function of REAL,REAL st
c > 0 & d > 0 & a1 < a2 &
f = ( d (#) TrapezoidalFS (a1−c,a1,a2,a2+c) ) | ['a1−c,a2+c']
holds f is_integrable_on ['a1−c,a2+c'];

theorem :: FUZZY_7:37
for a,b,c be Real, f,g be Function of REAL,REAL,
h be PartFunc of REAL,REAL st
a <= b & b <= c & f is continuous & g is continuous &
h | [.a,c.] = (f | [.a,b.]) +* (g | [.b,c.]) & f.b = g.b & [.a,c.] c= dom h
holds h | [.a,c.] is continuous;

theorem :: FUZZY_7:38
for a,b,p,q be Real, f be Function of REAL,REAL st a <> p &
f = (AffineMap (a,b)|].−infty,(q−b)/(a−p).]) +*
     (AffineMap (p,q)|[.(q−b)/(a−p),+infty.[)
holds f is Lipschitzian;

theorem :: FUZZY_7:39
for a,b,c be Real, f,g,h be Function of REAL,REAL
st
a <= b & b <= c & f is continuous & g is continuous &
h | [.a,c.] = (f| [.a,b.]) +* (g | [.b,c.]) & f.b = g.b
holds
integral((id REAL) (#) h,['a,c'])
= integral((id REAL) (#) f,['a,b']) + integral((id REAL) (#) g,['b,c']);

theorem :: FUZZY_7:40
for r be Real, f,g be Function of REAL,REAL holds
r (#) (f +* g) = (r (#) f) +* (r (#) g);

theorem :: FUZZY_7:41
for a, b, c, d, r be Real st
 a < b & b < c & c < d holds
(((AffineMap ((1 / (b − a)),(− (a / (b − a))))) | [.a,b.])) +*
((AffineMap (0,1)) | [.b,c.]) +*
((AffineMap ((− (1 / (d − c))),(d / (d − c)))) | [.c,d.])
= TrapezoidalFS (a,b,c,d) | [.a,d.];

theorem :: FUZZY_7:42
for a, b, c, d, r be Real st
 a < b & b < c & c < d holds
TrapezoidalFS (a,b,c,d)
= (AffineMap (0,0) | (REAL \ ].a,d.[)) +* TrapezoidalFS (a,b,c,d) | [.a,d.];

theorem :: FUZZY_7:43
for r be Real, f be Function of REAL,REAL holds
(r (#) f) | A = r (#) (f | A);

theorem :: FUZZY_7:44
for r be Real, f be PartFunc of REAL,REAL st A c= dom f
holds
(r (#) f) | A = r (#) (f | A);

theorem :: FUZZY_7:45
for a, b, c, d, r be Real st
 a < b & b < c & c < d holds
```

(r (#) AffineMap (1 / (b − a),− (a /(b−a)))) | [.a,b.] +*
(r (#) AffineMap (0,1)) | [.b,c.] +*
(r (#) AffineMap ((− (1 / (d − c))),(d / (d − c)))) | [.c,d.]
= (r (#) TrapezoidalFS (a,b,c,d)) | [.a,d.];

theorem *:: FUZZY_7:46 ::fuzzy618:*
for s **be** Real, f,g **being** Function **of** REAL,REAL **holds**
(f |].−infty,s.]) +* (g | [.s,+infty.[ )
is Function **of** REAL,REAL;

theorem *:: FUZZY_7:47*
for a,b,r **being** Real **holds**
r (#) (AffineMap (a,b)) = AffineMap (r*a,r*b);

theorem *:: FUZZY_7:48*
for a1,c,a2,d **be** Real, f **be** Function **of** REAL,REAL **st**
c > 0 **&** d > 0 **&** a1 < a2 **&**
f | [.a1−c,a2+c.] = AffineMap(d/c,−(d/c)*(a1 − c)) | [.a1−c,a1.] +*
    AffineMap(0,d) | [.a1 ,a2.] +*
    AffineMap(−d/c, (d/c)*(a2 + c)) | [.a2,a2+c.]
holds
integral(f,['a1−c,a2+c']) =
integral(AffineMap(d/c,−(d/c)*(a1 − c)), ['a1− c,a1']) +
integral(AffineMap(0,d) , ['a1 ,a2']) +
integral(AffineMap(−d/c, (d/c)*(a2 + c)) , ['a2,a2+c']) ;

theorem *:: FUZZY_7:49*
for a1,c,a2,d **be** Real **st** c > 0 **&** d > 0 **&** a1 < a2 **holds**
centroid (d (#) TrapezoidalFS (a1−c,a1,a2,a2+c), ['a1−c,a2+c']) = (a1+a2)/2;